# Targeted advertising with consumer search: an economic analysis of keywords advertising

Alexandre De Cornière

PARIS SCHOOL OF ECONOMICS
ECOLE D'ECONOMIE DE PARIS

WORKING PAPER N° 2009 - 48

# Targeted advertising with consumer search:

# An economic analysis of keywords advertising

Alexandre de Cornière

JEL Codes: D43, D83, L13, M37
Keywords: Search-engine, targeted advertising, consumer search, product differentiation

PARIS-JOURDAN SCIENCES ECONOMIQUES
LABORATOIRE D'ECONOMIE APPLIQUÉE - INRA

48, BD JOURDAN – E.N.S. – 75014 PARIS
TÉL. : 33(0) 1 43 13 63 00 – FAX : 33 (0) 1 43 13 63 10
www.pse.ens.fr

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE – ÉCOLE DES HAUTES ÉTUDES EN SCIENCES SOCIALES
ÉCOLE NATIONALE DES PONTS ET CHAUSSÉES – ÉCOLE NORMALE SUPÉRIEURE

# Targeted Advertising with Consumer Search: an Economic Analysis of Keywords Advertising

Alexandre de Cornière*

November 24, 2009

**Abstract**

This article investigates the role of a search engine as an intermediary between firms and consumers. Search engines enable firms to target consumers who have revealed some specific needs through their query. In a framework with horizontal product differentiation, imperfect product information and in which consumers incur search costs, I show that introducing a "neutral" targeted advertising mechanism reduces social inefficiencies and tends to reduce the equilibrium price. Moreover, the accuracy of the mechanism has a non monotonic effect on the price of the good: the price is lowest when the accuracy is intermediate.

**Keywords**: search-engine, targeted advertising, consumer search, product differentiation.
**JEL Classification**: D43, D83, L13,M37.

*Paris School of Economics, Email: adecorniere@gmail.com.

# 1 Introduction

In 2007, online advertising expenses amount to 21 billion dollars in the United States, which is about 7% of total advertising expenses (Evans (2008)). The main actors in this industry are the internet search engines, such as Google or Yahoo. Indeed, 40 percent of online advertising is search-related. Moreover, search-related advertising expenses have been multiplied by seven between 2002 and 2006.

It turns out that advertising through a search engine is the cheapest way of attracting new consumers (see Batelle (2005)). One may wonder what are the ingredients that make it so profitable. Two aspects seem to be of particular importance, namely the facts that (i) advertising is *intent-related* and (ii) costs are paid on a *per click* basis.

Intent-related advertising, as opposed to content-related advertising, exploits the possibility to know what consumers are looking for. Typically, when a consumer enters keywords such as "ink jet photo printer" on a search engine, he or she reveals a need, and firms which can satisfy this need are able to target this consumer, instead of having to rely on less-relevant characteristics of the audience which would be used with more traditional advertising, such as TV or magazines.

The other ingredient, the "per click" pricing, is aimed at ensuring announcers that their investments are not wasted, i.e that the consumers for whom they pay are those who actually see the ad *and* were looking for it.

In this paper I present a model of targeted advertising through a search engine, with differentiated products, which includes the main features mentioned above. Firms are uniformly distributed around a circle, and consumers do not have prior knowledge of firms' prices or positions on the circle. The search engine is an intermediary between firms and consumers: announcers choose which keywords they want to target, and consumers enter keywords and then search sequentially (and costly) at random through the links that appear. I do not study the format of the auction through which slots are allocated[1]. Rather, I shall explore the links between what information is revealed by the search engine and the resulting market outcomes.

In sections 2, 3 and 4, the search engine is "neutral", in the sense that it does not modify the messages which it receives. I compare the outcome with a situation in which there is no

[1] The per-click cost is determined through a *Generalized Second Price Auction* (See Edelman, Ostrovsky, and Schwarz (2007), Varian (2007), for the properties of this mechanism)

intermediary through which firms can target consumers. Basically, I find that consumers benefit from firms' ability to target through three channels: better matches, smaller expenses in search costs and lower prices than without targeting. The fact that consumers find products more suited to their tastes is rather in line with the intuition that one may have before going into the details of the model. Indeed, since announcers target them, consumers no longer receive non-relevant advertisements and thus choose from a better pool of offers. The model also predicts that, with targeting, consumers do not visit more than one firm, and thus minimize their search costs. These two results combine to improve the efficiency of advertising: the social costs due to imperfect information (bad matches and high search costs) are significantly reduced and thus the presence of a search engine contributes to improving social welfare.

Consumers are the main beneficiaries of this welfare improvement, for they also benefit from a lower price of the final good. To grasp the intuition of this result, it is useful to emphasize that in the model consumers actively search for goods. This search process is sequential: after learning an offer, a consumer compares this offer to the expected offer that he is going to receive if he continues searching (his "outside option"). If the difference between the outside option and the current offer is larger than the search cost, then the consumer continues searching. Now, when firms can target consumers, the relative quality of the outside option increases, because consumers know that the offers they will get after rejecting the current one are targeted at them, and thus very likely to be good matches. Thus, since firms essentially compete against outside options, a rise in the quality of the latter implies less bargaining power for the firms and thus a lower price for the final good.

The "neutral" matching technology is an approximation of how search engines really proceed. For instance, Google sorts announcers using a weighted average of the firms' bids and of a "quality score" index. Consumers are also sometimes provided with additional information on the results page, such as a map showing the locations of different vendors. On the other hand, the "Broad match" technology enables search engines to expand the set of keywords corresponding to a given advertisement. Such practices may be regarded as an attempt by the search engine to influence the accuracy of the information transmitted by firms, in one way or another. In section 5 I look at a situation in which the search engine can introduce an arbitrary level of noise (in a sense made precise below) in the information revealed to consumers. The analysis reveals that the equilibrium price is a non-monotonic function of the level of noise. For low levels of noise, firms behave like monopolies, whereas the competitive pressure is higher for intermediate levels of noise and decreases thereafter. The reason for this switch from monopoly-like equilibrium to oligopoly-like equilibrium lies in the relative importance of the two constraints that a firm's offer has to satisfy with respect to its customers: an individual rationality (IR) constraint (consumers must be better-off if they buy than if they leave the market) and an outside-option (OO) constraint (consumers must be better-off if they buy than if they continue searching).

- *Related literature*

This paper is related to the large literature on search models and advertising, as well as to more recent contributions which study internet search engines.

The literature on search models on a product market has provided important insights. In a seminal paper, Diamond (1971) shows that as soon as there is a positive cost for consumers to learn the price of a homogenous good, the only equilibrium outcome is for all the firms to charge the monopoly price. This result is known as the "Diamond paradox". Stahl (1989) studies situations in which consumers have different search costs. This heterogeneity implies that some consumers will be better informed than others. As in Varian (1980), the fact that consumers differ in their level of information generates equilibrium price dispersion, because some firms want to compete for the informed consumers (i.e. with low search costs) whereas other firms charge high prices and sell only to the uninformed consumers (i.e. with high search costs).

When products are differentiated, the price is an increasing function of the search cost and entry is generally excessive with respect to the social optimum (Anderson and Renault (1999), Wolinsky (1984)). Bakos (1997) studies the impact of a drop in search costs due to the development of electronic markets. He highlights the importance of the nature of the information which is costly to get: prices go down when consumers have a cheaper access to price information, whereas prices rise when it is easier to get product information.

The relationship between advertising and consumer search is not a new topic: in Robert and Stahl (1993), consumers may learn the price of homogenous goods either by receiving an ad or by searching actively. In a monopoly framework with uncertainty regarding the product's characteristics, Anderson and Renault (2006) study the optimal content of advertising. They highlight the differences between product information and price information, and show that the optimal advertising content varies with consumers' search costs.

The issue of targeting has received rather little attention in the economic literature. Esteban, Gil, and Hernandez (2001) show that in a monopoly framework, firms' ability to target consumers reduces both consumers' and total surplus. Iyer, Soberman, and Villas-Boas (2005) study targeting in a duopoly. Targeting induces endogenous differentiation of products, since firms advertise less to consumers who do not have "strong" preferences. The average price thus goes up. In their model, targeted advertising is more valuable to firms than targeted pricing. Also, interestingly, the effect of targeting on the optimal level of advertising depends on the initial cost of wasted advertising.

Van Zandt (2004) deals with the issue of information overload. He shows that, when firms can target consumers, a rise in the cost of advertising induces firms to send more accurate information to consumers, and this alleviates the effects of information overload.

Some recent papers study the interactions between firms and consumers on a search engine, but focus more on the ranking of ads than on the choice of relevant keywords. Athey and Ellison (2007) show that there exists an equilibrium in which efficient firms get the higher slots, and in which consumers search sequentially from top to bottom. They discuss mechanisms which could improve the efficiency of the generalized second-price auction.

Armstrong, Vickers, and Zhou (2009) study the impact of prominence on the market outcome. A prominent firm is sampled first by all consumers. Interestingly, they show that when firms are symmetric, prominence reduces welfare. On the other hand, when firms are vertically differentiated, firms with better quality would be willing to pay more to be made prominent, while consumers would sample these firms first even if they did not have to. Making the best firm prominent would improve welfare. This underlines the force that drives "better" firms to bid aggressively in order to secure the best slots, even when pricing is endogenous.

To the best of my knowledge, this paper is the first to explicitly model the transmission of information from firms to consumers through a search engine, and how this process may affect prices and welfare. It is also the first to study a model of consumer search with targeted advertising.

# 2 The model

- *Description of the market and of preferences*

The framework is based on Wolinsky (1983). Consider a market where a continuum of mass $\mu_F$ of firms (or "announcers") produce a differentiated good at a zero marginal cost. Each product may be described by a single keyword. Keywords are located on a circle, whose perimeter is normalized to one. Thus a firm is characterized by the position of its product's keyword on the circle. Keywords' positions are denoted by $x \in [0; 1]$.

There is a continuum of mass $\mu_C$ of consumers, each one having a favorite, or ideal, brand (or keyword), $y \in [0; 1]$.

Consumers have use for at most one product, and the utility that a consumer $y$ gets from consuming a good located in $x$, with $d(x, y) = d$, is

$$u(d, p) = v - td - p \tag{1}$$

where $p$ is the price of the good and $t$ is a transportation cost. $t$ actually measures the intensity of tastes : as $t$ goes to 0, consumers regard goods as being homogenous, whereas as $t$ increases consumers pay more attention to the products' attributes.

Consumers have imperfect information about firms' characteristics: they do not know firms' position on the circle nor their price, and thus have to search before buying.

- *Advertising technology*

Interactions between firms and consumers are only possible through a search engine. The search engine plays the role of an intermediary: firms communicate the set of keywords that they want to target, and consumers communicate the keyword they are interested in. Consumers cannot enter several keywords at the same time. If a certain keyword is entered by a consumer, all the firms who want to target this keyword appear on the consumer's screen. Consumers do not observe neither the prices nor the positions on the circle of any firm before they click on their link. If a consumer clicks on a firm's link, he incurs a search cost $s \in (0; t/4)$. This search cost corresponds to the time spent in order to find the relevant information on a website. On the other hand, when a consumer clicks on an announcer's link, the announcer pays an exogenous fee $a > 0$ to the search engine.

The assumption that consumers do not observe anything before clicking on a link seems appropriate in many contexts. Indeed, announcers can provide very little information with the text under their link on a search engine's page. Consumers have to click on the link to get more precise information. In this respect, advertising is not informative in the usual sense: it does not provide information in itself, but in equilibrium consumers infer correctly that a firm which targets them is not farther than a certain distance.[2].

After a consumer has sampled a firm and learned its price and position, he can come back at no cost (recall is costless). It is the case if for instance consumers open a new window every time they click on a link.

- *Strategies and equilibrium concept*

A strategy for a firm $x$ consists in the choice of a price $p$ and in a set of keywords $S = [x - D; x + D]$[3].

Consumers's strategy consists in choosing an optimal stopping rule, that is in setting a reservation distance $R$, such that the consumer is indifferent between buying a product at a distance $R$ and continuing to search. $R$ depends on the price that the consumer observes ($p$) as well as the strategy that he expects firms to play ($\overline{\sigma} = (\overline{p}, \overline{D})$). Thus I will use the notation $R(p, \overline{\sigma})$ to describe the stopping rule. The optimality of such a strategy is discussed at length in Stahl (1989) and Anderson and Renault (1999). Basically, when recall is costless, as long as there is at least one firm left to visit, the problem faced by the consumer is stationary and he cannot do better than searching sequentially using a stopping rule.

The equilibrium concept used is the perfect bayesian equilibrium: every firm sets its price and advertising policies so as to maximize its profit given the other firms' strategies and the stopping rule used by consumers. The stopping rule is itself a best-response to firms' strategies. When a consumer observes an out-of-equilibrium price, his belief about other firms' strategies does not change.

[2]The assumption is less relevant when consumers have a previous knowledge of the firms and/or products (if they bought in the past, of if they know the brand). I assume away these kinds of situation, which certainly deserve a proper analysis

[3]One could imagine a richer strategy space regarding the set of keywords. As a matter of fact, a richer strategy space would not destroy the equilibrium of this simpler game, even though there might be other equilibria using more complex strategies

# 3 Equilibrium analysis

The first proposition, whose proof is in the appendix, enables us to restrict the analysis to situations in which firms play pure strategies.

**Proposition 1** *There is no symmetric equilibrium of the game in which firms play strictly mixed strategies (i.e. in which at least two different actions are played with positive probability).*

Let $(p^*, D^*, R^*)$ be the equilibrium strategies. Consumers' and firms' strategies need to be a best-response to this strategy profile.

- *Optimal stopping rule*

In equilibrium, when a consumer $y$ clicks on a link, the expected utility he gets from this click if he buys is

$$\int_{y-D^*}^{y+D^*} (v - td(x,y) - p^*) f(x) dx = 2\int_0^{D^*} \frac{v - t|z| - p^*}{2D^*} dz$$

Consumers regard each click as a random draw of a location $x$ from a uniform distribution, whose support is $[y - D^*; y + D^*]$ and whose density is $f(.)$. Indeed a firm located at a distance greater than $D^*$ from $y$ would not appear on the results' page in equilibrium (the consumer would not be targeted). Suppose for now that all firms set the equilibrium price $p^*$. Then, after the first visit, the only way a consumer can improve his utility is by finding a closer firm. For $R^* \equiv R(p^*, \sigma^*)$ to be a reservation distance it must be such that a consumer is indifferent between continuing to search and buying the product:

$$2\int_0^{R^*} \frac{t(R^* - |z|)}{2D^*} dz = s \tag{2}$$

The left-hand side of this equality is the expected improvement if a consumer decides to keep on searching after being offered a product at a price $p^*$ and at a distance $R^*$. This expected improvement equals the search cost, so that the consumer is indifferent between buying or searching again. After a simple calculation one gets

$$R^* = \sqrt{\frac{2D^* s}{t}} \tag{3}$$

One may notice that the equilibrium reservation distance is independent of the equilibrium price. This is because so far I have ignored the individual rationality constraint, $v - td - p \geq$

0. We will see that this constraint is always satisfied in equilibrium. $R^*$ is an increasing function of the equilibrium reach of advertising $D^*$: if consumers expect firms to try to reach a wide audience (by targeting many keywords), they adjust their stopping rule by being less demanding, because the expected improvement after a given offer is lower than with more precise targeting.

Now, when a consumer samples a firm which has set an out-of-equilibrium price $p \neq p^*$, I assume that his belief about other firms' strategy does not change, and therefore his optimal stopping rule $R(p, \sigma^*)$ is such that accepting a price $p$ at a distance $R(p, \sigma^*)$ gives the same utility as accepting a price $p^*$ at a distance $R^*$, i.e $v - tR(p, \sigma^*) - p = v - tR^* - p^*$. Thus we have the following proposition.

**Proposition 2** *Given other firms' expected strategy $\sigma^* = (p^*, D^*)$, a consumer accepts to buy a good at price p if and only if the selling firm is located at a distance less than $R(p, \sigma^*)$, with $R(p, \sigma^*)$ such that*

$$R(p, \sigma^*) = R^* + \frac{p^* - p}{t} = \sqrt{\frac{2D^*s}{t}} + \frac{p^* - p}{t}$$

- *Optimal advertising and pricing strategies*

Suppose that firm $x$ sets a price $p$. Since it only has to pay for consumers who actually visit its link, firm $x$'s optimal strategy is to appear to every consumer $y$ such that the expected profit made by $x$ through a sale to $y$ conditionally on $y$ clicking on $x$'s link is positive, i.e

$$Pr(y \text{ buys } x\text{'s product} | y \text{ clicks on } x\text{'s link}) \times (p - a) \geq 0 \tag{4}$$

where $a$ is the per-click fee paid to the search engine.

The next lemmas will enable us to derive the only symmetric equilibrium. At this equilibrium, every firm chooses to advertise only to the consumers who buy the product as soon as they click on its link. Thus no consumer visits more than one firm.

The first lemma gives a necessary condition satisfied by any symmetric equilibrium.

**Lemma 1** *Any symmetric profile of strategy $\sigma = (p, D)$ such that $D \neq R(p, \sigma)$ cannot be an equilibrium.*

*Proof*: This proof is in two stages: (1) if firms set $D < R(p, \sigma)$, then a firm can profitably deviate targeting more consumers (2) if $D > R(p, \sigma)$, there is always at least one firm which can profitably deviate and lower its targeting distance.

1. The first stage is rather straightforward: suppose that all firms have a targeting distance $D$ smaller than $R(p, \sigma)$. Take a consumer $y$ and a firm $x$ such that $D < d(x, y) < R(p, \sigma)$. If $x$ were to deviate and choose to appear to consumer $y$, then it would sell the good with probability one if $y$ clicked on its link. Thus it would be a profitable deviation.

2. Now suppose that all firms set $D > R(p, \sigma)$. Take a consumer $y$, and denote $\overline{x}$ the firm which is located farthest away from him. Since $d(\overline{x}, y) > R(p, \sigma)$, the probability that $y$ buys from $\overline{x}$ is zero. By reducing its reach, firm $\overline{x}$ can improve its profit.

□

Therefore, if a symmetric equilibrium exists, it must be the case that firms choose a targeting distance equal to consumers' equilibrium reservation distance. The next step in order to derive a symmetric equilibrium of the game is to study the best response of a firm when other firms play a symmetric strategy $\sigma^* = (p^*, D^*)$ with $D^* = R(p^*, \sigma^*)$.

**Lemma 2** *Let $x$ be the location of a given firm on the circle. If:*

- *all the other firms play the strategy $\sigma^* = (p^*, D^*)$ where $D^* = R(p^*, \sigma^*)$, and*
- *consumers expect all firms to play $\sigma^* = (p^*, D^*)$ and thus play $R(p, \sigma^*) = \sqrt{\frac{2sD^*}{t}} + \frac{p^* - p}{t}$,*

*then, whatever price $p$ firm $x$ decides to set , the optimal advertising strategy is to set $D(p) = R(p, \sigma^*)$, i.e. a targeting distance equal to the reservation distance of consumers who face an "out of equilibrium" price.*

This lemma states that if a firm wants to deviate from a situation where all firms set targeting distance equal to the "equilibrium" reservation distance, the deviation implies to set a scope of relevance equal to the "out of equilibrium" reservation distance. Thus, the deviation does not change the number of clicks per consumer, since they find it optimal to buy from the first firm they visit. The proof is very similar to the previous lemma's one, and is omitted.

Thanks to Lemma 2, it is straightforward to compute the optimal strategy of a firm. Given that the other firms play $D^* = \frac{2s}{t}$ (which is obtained by solving $D^* = R(p^*, \sigma^*)$), and given that $D(p) = R(p^*, \sigma^*) = \frac{2s+p^*-p}{t}$, firm $x$'s profit is equal to $2\frac{\mu_C}{\mu_F} D(p)(p-a)$ that is

$$\pi(p) \propto 2(p-a)\frac{2s+p^*-p}{t}$$

Notice here that $a$ plays the role of a marginal cost: since consumers buy at their first visit, each firm pays $a$ exactly the same number of times as it sells the product. Firm $x$'s best response to the equilibrium strategy is therefore $p^{BR}(p^*) = \frac{2s+p^*+a}{2}$. For $p^*$ to be an equilibrium, it must be the case that $p^* = p^{BR}(p^*)$, i.e

$$p^* = 2s + a \tag{5}$$

The equilibrium strategies are summarized below:

**Proposition 3** *There exists a unique symmetric equilibrium in pure-strategy.*

- *Firms set a price equal to* $p^* = 2s + a$
- *They target all the keywords located at a distance less than or equal to* $D^* = \frac{2s}{t}$
- *Consumers buy whenever they find a firm at a distance less than or equal to* $R(p, \sigma^*) = \frac{2s+p^*-p}{t}$

In equilibrium, as a corollary, consumers always buy from the first firm they visit.

# 4 Comments on the equilibrium

- *Some comparative statics*

Some results of the preceding analysis deserve particular attention. Regarding the level of advertising, as measured by the equilibrium reach of advertising $D^*$, we see that it is an increasing function of the search cost $s$ and a decreasing function of the transportation cost $t$. This is in line with the intuitive signification that one may give to these parameters. Indeed, $s$ and $t$ are both a source of market power for the firms, but of a different nature. It may be convenient to regard the strategic interactions as a bargaining process in which the firm makes a take-it-or-leave-it offer to a consumer who has some private information about his type. The consumer has an outside option, which is to visit another firm. A rise in the search cost $s$ strengthens a firm's bargaining position with respect to *all the consumers who have just clicked on its link*, because the outside option is less attractive. This applies to every consumer who has clicked, no matter how far he is from the firm.

On the other hand, a rise in the transportation cost $t$ does not affect all the firm-consumer relationships the same way. Intuitively, if a consumer is close from the firm which makes him the offer, a rise in $t$ implies that the consumer pays more attention to the distance between him and the firm, and thus he is more likely to accept, other things being equal (in particular the value of the outside option). But if the distance between the consumer and the firm is greater, a rise in $t$ makes the consumer more reluctant to buy, other things being equal. Thus we see that a rise in $t$ improves the firms' bargaining power *vis-à-vis* close consumers but deteriorates market power *vis-à-vis* distant consumers.

Having said that, it is straightforward to see why $s$ and $t$ have opposite effects on the equilibrium advertising level. A rise in $s$ makes distant consumer more willing to buy, and thus the firm wants to target them, and inversely for a rise in $t$.

The above reasoning does not explain why the transportation cost $t$ does not have any effect on the price level $p^* = 2s + a$.

Basically, a rise in $s$ makes the offer more attractive to consumers who click on it, and, since announcers expand their reach, makes the outside option less valuable. Both effects improve firms' bargaining power, and thus lead to higher prices.

A rise in $t$ has a more ambiguous effect: it improves the bargaining power of the firm vis-à-vis close consumers, deteriorates bargaining power vis-à-vis distant consumers, and the firm advertises less. These effects tend to push the price up. But there is another, more subtle, effect: a rise in $t$ leads other firms to reduce the reach of their advertising, and therefore *improves* the outside option of *all* the consumers, because they expect that the next firm they visit is at a distance smaller than if $t$ was lower. In the model with linear transportation costs, these two effects offset each other and therefore firms do not benefit from more differentiation. The result that $p^*$ does not depend on $t$ is not robust to a change in the form of the transportation cost, but introducing other functional forms (e.g. quadratic transportation costs) would not alter the insight that targeted advertising improves consumers' bargaining power through a better outside option.

Regarding the reservation distance $R(p, \sigma^*) = \frac{2s+p-p^*}{t}$, the effects are roughly the same as above. The reservation distance raises with $s$, as the outside option is less valuable. It is a decreasing function of $t$.

- *A useful benchmark*

In order to correctly assess the impact of targeted advertising on market outcomes, it is useful to compare the results obtained above with results which would obtain if firms did not have the ability to target consumers.

This may be done by using Wolinsky 1983's model. The fundamental difference between that model and the model with targeting is that, in Wolinsky's model, each consumer receives all the advertisements, i.e it is as if firms targeted the whole circle. That model is therefore a benchmark which tends to underestimates the positive effects of targeting for consumers. Indeed, if one prevents targeting, the best thing for consumers is to receive all the ads and search sequentially.

In the linear version of Wolinsky's model, consumers' reservation distance writes $R_W = \sqrt{\frac{s}{t}}$, and the equilibrium price is $p_W = \sqrt{st}$. The average number of visits per consumer is $1/R_W = \sqrt{\frac{t}{s}} \geq 2$. The average distance between a buyer and a seller is $R_W/2$, and thus the average consumer utility is $u_W v - \frac{5}{2}\sqrt{st}$ How do these findings compare to the model with targeting? To facilitate the comparison, let the advertising cost $a$ tend to zero.

First, the price with targeting is lower, since $2s \leq \sqrt{st} \Longleftrightarrow s/t \leq 1/4$, which is true [4]. This result is different from results obtained by Iyer, Soberman and Villas-Boas (2005) and Esteban, Gil and Hernandez (2001) which obtain in duopoly or monopoly framework respectively. In Iyer, Soberman and Villas-Boas (2005)'s model, targeted advertising enables firms to differentiate: consumers with strong preferences for one product are not targeted by the other firm, and therefore firms are in local monopoly. In Esteban, Gil and Hernandez (2001), the monopolist faces a less elastic demand with targeting and is therefore able to raise its price. In my paper, these effects are offset by an improvement of the outside option of the consumer, and therefore the elasticity of demand is raised, which leads to a lower price.

[4] otherwise $\frac{2s}{t}$ would be larger than $1/2$, and this would not make much sense in the model with targeting

Consumers' reservation distance is higher without targeting, due to the low value of the outside option: if a consumer refuses an offer, the next offer he receives is a random draw uniformly distributed around the circle, instead of a random draw from an interval around his position. A direct consequence is that the average distance (which equals $R/2$ in both models) is also lower with targeting, which implies that targeting improves efficiency on the ground of better matching.

Targeting also reduces the number of visits before a purchase. Indeed, although the reservation distance is higher without targeting, it is still smaller than 1/2, which implies that some consumers will receive offers that they do not accept in equilibrium.

One may also see that the differentiation parameter $t$ has a positive effect on the price: $p_W = \sqrt{st}$. In light of the previous comments on the effects of $t$, the reason is simple: a rise in $t$ does not affect the value of the outside option, because the offers are drawn from the same distribution. Thus the positive differentiation effect on the mark-up is not offset by the "outside option effect".

# 5 Accuracy of the matching process

In this section I turn to the question of the amount of information revealed by the search engine. In the basic model, no "hard" information is revealed to consumer regarding firms' positions on the circle. In equilibrium consumers anticipate correctly that firms are somehow close to them, but they have no other information.

The actual system is a bit different, in the sense that ads are sorted on the screen of a consumer. The sorting of ads is done by the search engine, on the basis of the announcers' bids and of a so-called "quality score". The way the quality score of an announcer is computed is unclear. Google, for instance, only gives some of the factors that are used to compute it[5]:historical clickthrough-rate (which measures the number of clicks generated by ads from a given announcer), relevance to the query, quality of the landing site, among other factors.

How would the conclusions of the model be affected by the introduction of a quality score? To see this this, I deal with a very simple proxy for the quality score, namely the position on the circle: the search engine reveals firms' positions on the circle, so that consumers can choose

[5]This information may be found at http://adwords.google.com/support/bin/answer.py?answer=10215

which one to visit. Nevertheless I still assume that the search engine cannot observe the price set by firms. To simplify matters even more, I restrict firms' strategy space to the set of prices: firms do not choose which keywords to target. On the other hand, consumers' strategy space now includes the *choice* of firms to visit.

In this slightly modified framework, the only equilibrium is such that firms hold-up consumers and set a very high price.

To see this, suppose that consumers expect firms to set a price $p^*$. We need to find which firms a given consumer $y \in [0; 1]$ will visit , as well as his stopping rule. Since he anticipates that all firms set the same price, he strictly prefers to visit the firm which is the perfect match for him, i.e $x = y$.

Now, if firm $x$'s price is $p \leq p^*$, he stops searching and buys. But if $p > p^*$, he faces a trade-off between buying at a high price ($p$) and paying a search cost in order to buy at a lower price ($p^*$) from a slightly less satisfying firm (from his point of view). Since there is a continuum of firms, the difference in positions between two firms can be made arbitrarily small, and thus the consumer buys the product at price $p > p^*$ if and only if $p \leq p^* + s$.

We recognize the classical hold-up problem (see Diamond (1971)): knowing how consumers behave, the only symmetric equilibrium is such that $p^* = v$. Indeed, suppose that $p^* < v$ is the price set by all firms. Then any firm can profitably deviate by setting a price equal to $p^* + s$, since at that price the consumers who visit the firm buy from it.

This equilibrium is thus such that firms get all the surplus from trade. But, as the reader may have anticipated, this is not individually rational for a consumer to start searching, because he will incur the search cost $s$ and get zero surplus. Therefore the market collapses!

Although a bit extreme, this conclusion sheds light on a potential difficulty, namely that firms could benefit from a hold-up situation *vis-à-vis* consumers and that trade could be hampered to some extent. Revealing too much information to consumers can be damaging as long as this information is price-irrelevant. This result is very similar to a result in Bakos (1997) and the intuition is also present in Anderson and Renault (2000), although in a different set-up.

The situation above corresponds to a case in which the search engine chooses to impose $D = 0$ to firms, that is a case in which all noise has been removed from the sampling process. Let us look at the equilibrium when the search engine is able to choose the level of noise in the sampling process, i.e to choose arbitrary values for $D$.

This technology might be regarded as an approximation of the "broad match" technology which is used by Google to match queries and advertisements. Basically, with broad match, the search engine will display an advertisement even if the keyword has not been selected by the announcer, provided it is regarded as relevant by the search engine. For instance, suppose that an announcer selects only one keyword, namely "web hosting". If a consumer enters the keyword "web hosting company" or "webhost", then the announcer's advertisement will appear on the consumer's screen. Google argues that one of the benefits brought by such a practice is that it saves time for announcers: they no longer have to spend time and resources finding exactly what are the right keywords to use. The search engine will do that for them, using the available information on past queries and results in order to find relevant keywords.

Suppose that the search engine chooses the advertising distance $D$, everything else being unchanged. Now firms' strategy consists only in setting a price.

Consumers' reservation distance is still $R(p, \overline{p}, D) = \sqrt{\frac{2sD}{t}} + \frac{\overline{p}-p}{t}$, with $p$ being the observed price and $\overline{p}$ the expected price set by other firms.

**Proposition 4** *Suppose that the search engine sets a broad match distance equal to $D$, and that $v > 4s$.*

- *If $D < \frac{2s}{t}$, then the equilibrium price is $p^* = v - tD$.*
- *If $D = \frac{2s}{t}$, there is a continuum of equilibrium prices on the interval $[2s; v - 2s)$.*
- *If $D > \frac{2s}{t}$, then the equilibrium price is $p^* = \sqrt{2stD}$.*

The proof of this proposition is in the appendix.

As one can see on Figure 1, the price is a non-monotonic function of the degree of targeting $D$. When $D < \frac{2s}{t}$, we are in the case in which $D < R(p^*, p^*, D)$. The marginal consumer (in the sense that he is the farthest that a firm may reach) strictly prefers to buy the product than to search again. Thus firms act as if they had no competitors on the entire segment of length $2D$. The binding constraint is $v - tD - p \geq 0$ for the marginal consumer, who is at a distance $D$. Every firm acts as a monopoly and captures the marginal's consumer surplus. [6] This is a variation of the Diamond paradox.

[6] Assuming $v > 4s$ ensures that the firm who acts like a monopoly still wants to serve all consumers. The case $v \leq 4s$ is uninteresting since consumers would have a negative expected utility.

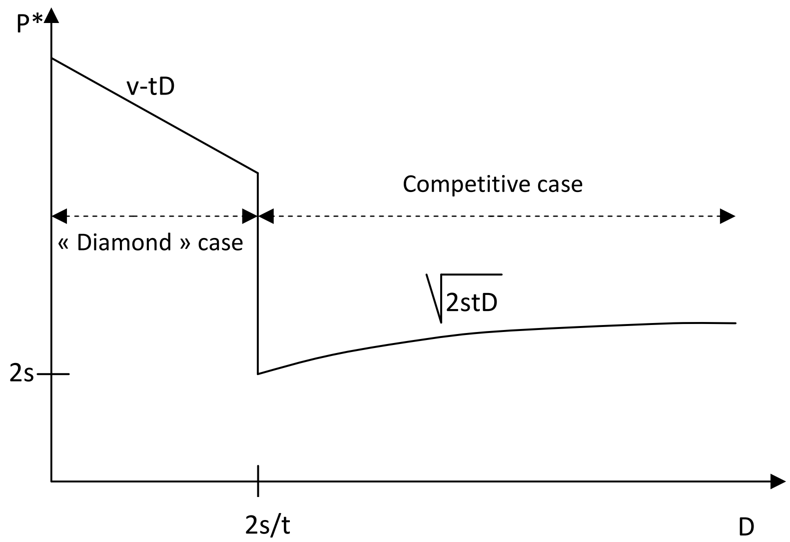


Figure 1: Equilibrium price with broad match

To put it differently, suppose that the other firms set a price $p_{-i}$. If $D < \frac{2s}{t}$, $p_{-i}$ is located on an inelastic portion of the demand curve of firm $i$. Therefore firm $i$ is not constrained by its competitors.

When $D = \frac{2s}{t}$, we have $D = R(p^*, p^*, D)$: if all the other firms set a price $p_{-i} \in [2s, v - 2s)$, firm $i$'s demand curve is inelastic up to $p_{-i}$, and elastic enough above $p_{-i}$ that the best response is to charge $p_{-i}$.

If $D > \frac{2s}{t}$, then $D > R(p^*, p^*, D)$. This implies that for a price superior or equal to $p_{-i}$, firm $i$ would not sell to all the consumers who visit it. Thus firm $i$ is constrained by its competitors, and we may label this situation as the competitive case.

Intuitively, when $D$ is small, each firm knows that it is sufficiently close to the consumers who visit it that none of them will want to search again. Firms act like monopolies. For intermediary values of $D$, this virtual isolation disappears: some consumers are now willing to switch to another firm if the price is too high. There is now an "outside option constraint" exerted by competitors, which leads the price to drop : firms can no longer act as if they were monopolies, and the price is thus the competitive price[7]. But as $D$ further increases, this outside option constraint becomes less stringent, because the average distance between a

[7]The competitive price is still above marginal cost, since competition is imperfect because of information frictions.

consumer and the next firm is larger, leading to a rise in the price.

One implication of Proposition 4 is that, when the advertising fee $a$ is small, the equilibrium outcome of the game in which firms choose their advertising strategy corresponds to the lowest possible equilibrium price of the game with exogenous targeting , namely $p^* = 2s$. The reason is that when firms choose their targeting strategy, no firm is willing to expand its reach farther than consumers' reservation distance, nor to reduce it below this distance. This implies that the equilibrium targeting distance $D^*$ is the smallest $D$ such that the outside option constraint is effective. A smaller $D$ would generate monopoly-like equilibrium, while a larger $D$ would make the constraint less stringent.

- *Possible extension*

Proposition 4 may have implications in terms of the optimal design of a matching mechanism. Suppose that the mass of consumers $\mu_C$ is an increasing function of their *ex ante* utility $F(U_C) = F(v - t\overline{d} - p^* - ns)$, where $\overline{d}$ is the average distance between a consumer and the product he eventually purchases, and $n$ is the average number of clicks. The profit of a search engine writes $\Pi_{SE} = anF(U_C)$. For $D > 2s/t$ there is a positive correlation between $n$ and $p^*$, and therefore increasing the number of clicks might also increase the equilibrium price, leading to a shrink in the number of users. This brief analysis is merely illustrative of the potential trade-off faced by the search engine. A possible direction for future research would be to incorporate this trade-off in a two-sided market framework (see Armstrong (2006) for instance) which would also take into account firms' entry and pricing issues.

# 6 Concluding remarks

Search engines allow intent-related targeted advertising, and this paper illustrates the potential efficiency gains generated from firms' ability to target consumers. An interesting effect is the fact that targeting improves consumers' outside options, and thus leads to a lower price. By choosing targeting accuracy, the search-engine may be able to affect the degree of competition between firms. Competition is more intense for intermediate values of targeting accuracy. This observation suggests that it may be possible for the search engine to design the matching mechanism in such a way as to generate more revenue, although this issue is left for further research.

# References


Anderson, S. P., and R. Renault (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 30(4), 719–735.

——— (2000): "Consumer Information and Firm Pricing: Negative Externalities from Improved Information," *International Economic Review*, 41(3), 721–42.

——— (2006): "Advertising Content," *American Economic Review*, 96(1), 93–113.

Armstrong, M. (2006): "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37(3), 668–691.

Armstrong, M., J. Vickers, and J. Zhou (2009): "Prominence and consumer search," *RAND Journal of Economics*, 40(2), 209–233.

Athey, S., and G. Ellison (2007): "Position Auctions with Consumer Search," Levine's Bibliography 122247000000001633, UCLA Department of Economics.

Bakos, Y. (1997): "Reducing Buyer Search Costs: Implications for Electronic Marketplaces," *Management Science*, 43(12), 1676–1692.

Batelle, J. (2005): *The Search: How Google and Its Rivals Rewrote the Rules of Business and Transformed Our Culture*. Nicholas Brealey Publishing.

Diamond, P. A. (1971): "A model of price adjustment," *Journal of Economic Theory*, 3(2), 156–168.

Edelman, B., M. Ostrovsky, and M. Schwarz (2007): "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords," *American Economic Review*, 97(1), 242–259.

Esteban, L., A. Gil, and J. M. Hernandez (2001): "Informative Advertising and Optimal Targeting in a Monopoly," *Journal of Industrial Economics*, 49(2), 161–80.

Evans, D. S. (2008): "The Economics of the Online Advertising Industry," *Review of Network Economics*, 7(3), 359–391.

Iyer, G., D. Soberman, and J. M. Villas-Boas (2005): "The Targeting of Advertising," *Marketing Science*, 24(3), 461–476.

Robert, J., and D. O. Stahl (1993): "Informative Price Advertising in a Sequential Search Model," *Econometrica*, 61(3), 657–86.

Stahl, D. O. (1989): "Oligopolistic pricing with sequential consumer search," *American Economic Review*.

Van Zandt, T. (2004): "Information Overload in a Network of Targeted Communication," *RAND Journal of Economics*, 35(3), 542–560.

Varian, H. R. (1980): "A Model of Sales," *American Economic Review*, 70(4), 651–59.

——— (2007): "Position auctions," *International Journal of Industrial Organization*, 25(6), 1163–1178.

Wolinsky, A. (1983): "Retail Trade Concentration Due to Consumers' Imperfect Information," *Bell Journal of Economics*, 14(1), 275–282.

——— (1984): "Product Differentiation with Imperfect Information," *Review of Economic Studies*, 51(1), 53–61.

# A Proof of Proposition 1

This proof may be skipped at the first reading. The reasoning is similar to some parts of the main text, and it is explained in greater details there.

Suppose that firms play a mixed strategy $\sigma$, in which $\sigma(p, D)$ is the probability with which a firm charges a price $p$ and advertises on a segment of length $2D$. Consumers compute their optimal stopping rule $R(p, \sigma)$ given this mixed-strategy. The stopping rule is such that consumers are indifferent between buying a product at price $p$ and at a distance $R(p, \sigma)$, given that firms play according to $\sigma$, and searching again. Let $\overline{D} = sup\{D, \exists ps.t.\sigma(p, D) > 0\}$. Let $p(D)$ be the average price set by a firm who chooses an advertising reach of $2D$, and $q(d)$ be the average price that a consumer expects to observe if the firm he faces is located at a distance $d$. Let $G(d) \equiv P_\sigma(D \leq d)$ be the marginal cdf of $D$ according to $\sigma$, and $g(d)$ the corresponding pdf. We have $q(d) = \frac{1}{1-G(d)} \int_d^{\overline{D}} p(x)g(x)dx$. A consumer is indifferent between buying at price $p$ and distance $R$ and searching again if and only if

$$\Phi(p, R) \equiv 2 \int_0^{\overline{D}} [t(R - x) + p - q(x)]^+ g(x)dx = s$$

The optimal stopping rule consists in setting a reservation distance $R(p, \sigma)$ such that $\Phi(p, R(p, \sigma)) = s$. A similar reasonning to lemmas 1 and 2 reveals that firms can do no better than setting $D(p) = R(p, \sigma)$: they may choose the price randomly, but once a price is chosen there is a unique optimal advertising strategy.

Thus the profit of a firm $\Pi(p)$ is proportional to $(p - a)R(p, \sigma)$. In order to have strictly mixed strategies in equilibrium, it is necessary for the set of solutions to the firm's profit maximization program to have more than one element. But this is impossible: $\Phi(p, R)$ is strictly increasing in both its arguments. If we raise $p$ by an amount $dp$, $R$ has to be reduced by an amount $dp/t$ to keep $\Phi(p, R)$ constant. Thus $R(p, \sigma)$ is linear in $p$: $R(p, \sigma) = c - p/t$. Therefore the maximization of $(p - a)R(p, \sigma)$ admits a unique solution, contradicting the assumption that firms play strictly mixed strategies.$\square$

# B Proof of Proposition 4

Suppose that a firm sets a price $p$ while all the other firms choose $p^*$. For a consumer located at a distance $d$ to buy from firm $x$, three conditions have to be satisfied: (i) $d \leq D$ ; (ii)$v - td - p \geq 0$; (iii) $d \leq R + \frac{p^*-p}{t}$. Condition (i) means that buyers have to see the advertisement. Condition (ii) ensures that it is individually rational for a buyer to buy the good at price $p$. Condition (iii) means that a consumer who is farther away from the firm than his reservation distance will not buy from this firm. Any consumer who satisfies the three conditions above will buy as soon as he clicks on the firm's link. Therefore the demand for a firm which sets a price $p$ is proportional to $min(D, \frac{v-p}{t}; \sqrt{\frac{2sD}{t}} + \frac{p^*-p}{t})$.

Now,

- $D \leq \sqrt{\frac{2sD}{t}} + \frac{p^*-p}{t} \iff p \leq p^* + \sqrt{2stD} - tD \equiv p_1$.
- $D \leq \frac{v-p}{t} \iff p \leq v - tD \equiv p_2$.

- $\frac{v-p}{t} \leq \sqrt{\frac{2sD}{t}} + \frac{p^*-p}{t} \Longleftrightarrow v \leq \sqrt{2stD} + p^*$.

Let $Q(p) \equiv min(D, \frac{v-p}{t}; \sqrt{\frac{2sD}{t}} + \frac{p^*-p}{t})$. If $p < min(p_1, p_2)$, then $Q(p) = D$. Thus there cannot be an equilibrium in which $p^* < min(p_1, p_2)$, because demand is price inelastic on this segment and thus firms would have an incentive to raise the price (see Figure 2 ).

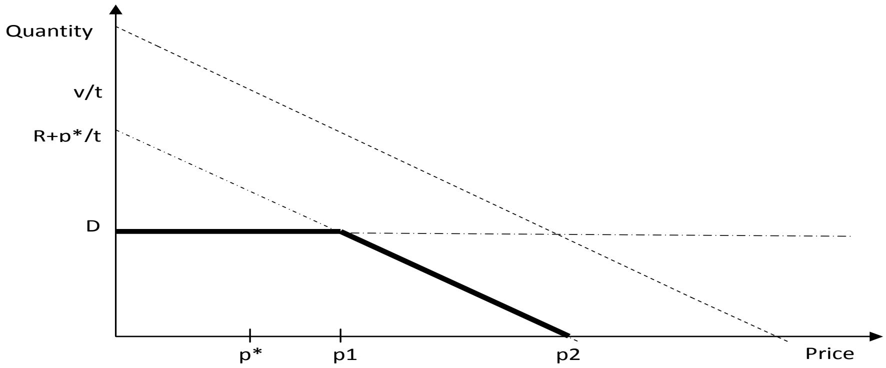


Figure 2: No equilibrium

Therefore any symmetric equilibrium must be such that $p^* \geq min(p_1, p_2)$. Now we must deal separately with cases according to whether $p_1 < p_2$ or $p_1 \geq p_2$.

## Case 1: $p_1 < p_2$ and $p^* > p_1$

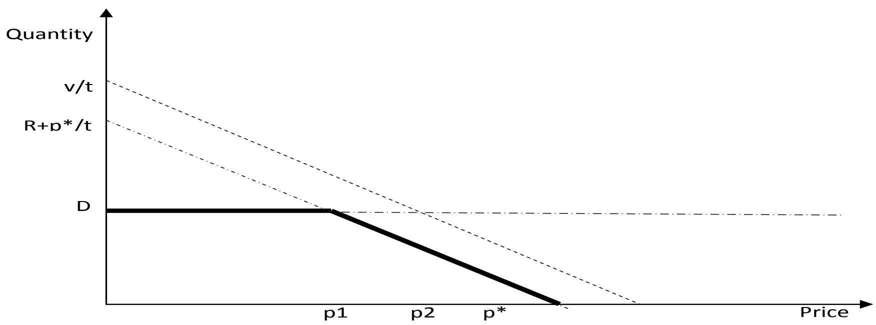


Figure 3: Quantity sold in case 1

We have $p_1 < p_2$ and $p^* > min(p_1, p_2)$ if and only if $\sqrt{2stD} + p^* < v$ (1.1) and $\frac{2s}{t} < D$ (1.2). As one can see on Figure 4 , the best response of a firm is to set the price equal to the maximum of $p_1$ and $\hat{p} \equiv argmaxp(\sqrt{\frac{2sD}{t}} + \frac{p^*-p}{t}) = \frac{\sqrt{2stD}+p^*}{2}$.

- We have $p_1 \leq \hat{p} \Longleftrightarrow p^* \leq 2tD - \sqrt{2stD}$ (1.3). If (1.1) ,(1.2) and (1.3) hold, the equilibrium price must be such that $p^* = \hat{p}$, that is $p^* = \sqrt{2stD}$. It is straightforward to check that this equilibrium price is

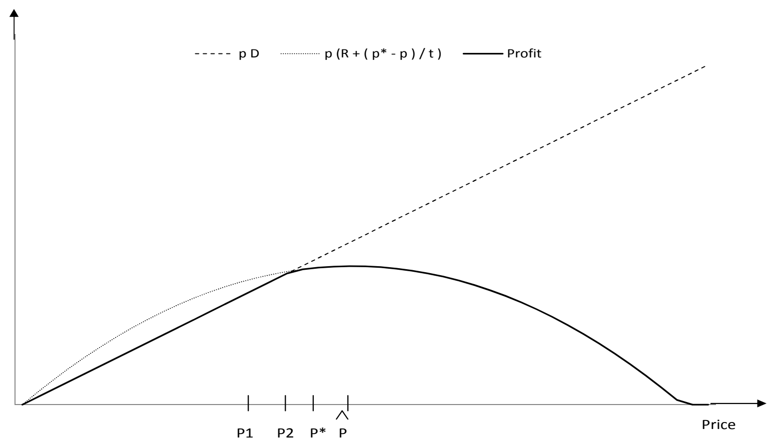


Figure 4: Profit function in case 1

consistent with (1.1) ,(1.2) and (1.3) if $v \geq 2\sqrt{2stD}$ and $\frac{2s}{t} \leq D$. Since the largest value of $D$ is $\frac{1}{2}$, a sufficient condition for this equilibrium to exist for every value of $D$ is $v \geq 2\sqrt{st}$. But this condition is met if we assume that the expected utility of a consumer in Wolinsky's model without targeting is non-negative. Even if this condition does not hold, this equilibrium exists when $\frac{2s}{t} \leq D \leq \frac{v^2}{8st}$ (notice that $\frac{v^2}{8st} \geq \frac{2s}{t} \Longleftrightarrow v \geq 4s$).

- If (1.3) does not hold, the best response of the firm is to charge $p_1$. The equilibrium price would be such that $p^* = p_1$, which is impossible since $\frac{2s}{t} < D$.

## Case 2: $p_1 \geq p_2$

We have $p_1 \geq p_2$ and $p^* \geq min(p_1, p_2)$ if and only if $\sqrt{2stD} + p^* \geq v$ (2.1) and $v - tD \leq p^*$ (2.2).
The best response of the firm is to set a price equal to the maximum of $p_2$ and $p^\sharp \equiv argmaxp(\frac{v-p}{t}) = \frac{v}{2}$.

- Suppose that $p_2 \geq p^\sharp$, that is $v - tD \geq v/2$ (2.3). In this case the candidate equilibrium price is $p^* = v - tD$. This price is always consistent with (2.2), but (2.1) holds if and only if $D \leq \frac{2s}{t}$.
- When (2.3) does not hold, the only possible equilibrium price is $p^* = \frac{v}{2}$. As before, (2.2) always holds, but (2.1) holds if and only if $D \leq \frac{v^2}{8st}$.

## Case 3: $p_1 < p_2$ and $p^* = p_1$

We are in case 3 if and only if $D = \frac{2s}{t}$ (3.1) and $p^* < v - tD$ (3.2).
If these two conditions are verified, the best response of the firm is to set a price equal to the maximum of $p_1$ and $\hat{p}$.

- We have $p_1 = p^* \geq \hat{p} \Longleftrightarrow p^* \geq \frac{\sqrt{2stD}+p^*}{2} \Longleftrightarrow p^* \geq 2s$ (3.3). When this holds, the best response of the firm is $p = p^*$. Thus any price $p^*$ such that (3.1), (3.2) and (3.3) hold is an equilibrium: when $D = \frac{2s}{t}$, there is a continuum of equilibrium prices on the interval $[2s; v - 2s)$ if $v \geq 4s$. If $v < 4s$, no price is such that (3.2) and (3.3) hold simultaneously.
- When (3.3) does not hold, the best response is $p = \hat{p}$, which leads to $\hat{p} = p^*$, i.e $p^* = 2s$, which is impossible if (3.3) does not hold.